Patterns of Attractors in the "Brain" - Wild Dynamics at the Edge

Enrique R. Pujals
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May 2012, Toronto

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1. Observing the task performed by the agent;
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3. the fact that the agent has been evolved by a Genetic Algorithm.

How to describe main dynamical properties

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General paradigm in the qualitative theory of Dynamical Systems

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We will try to suggest (CARTOON):

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Genetic algorithm as a form of searching algorithm
find a "good initial condition"

General framework. Why dynamics?

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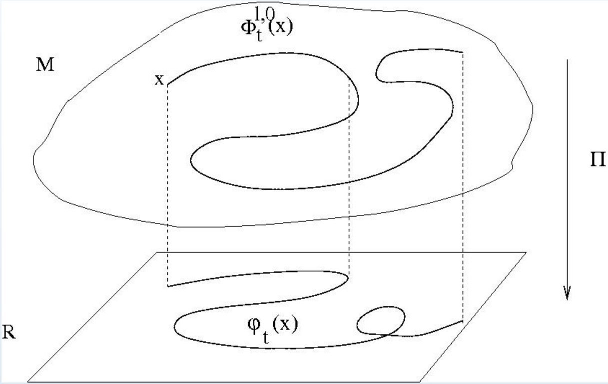

Why dynamics?

The ODE/neural network/“brain” is evolved by a “Genetic algorithm”

$$\mathcal{B}^1 = B_1^1, \ldots, B_n^1 \to \mathcal{B}^2 = B_1^2, \ldots, B_n^2$$

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Twofold approach:

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- Understanding the brain through its interaction with the environment and task;
- forced by an evolutionary process.

A few words about dynamical systems

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Pick up your preferred Differential Equation

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Mechanics,

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Mechanics, chemical reaction,

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Mechanics, chemical reaction, growth population,

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'Equation"

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‘Equation” $\boxed{\dot{x} = F(x)}$

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This is a dynamical system

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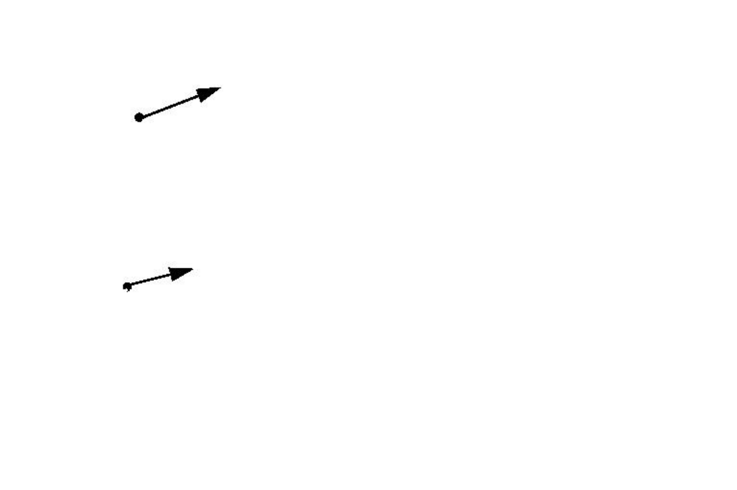

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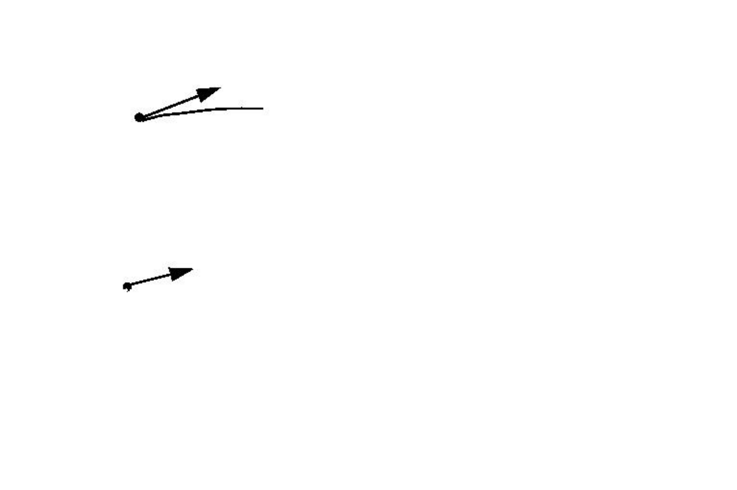

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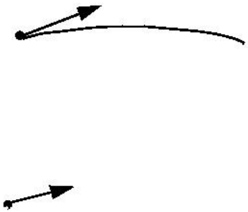

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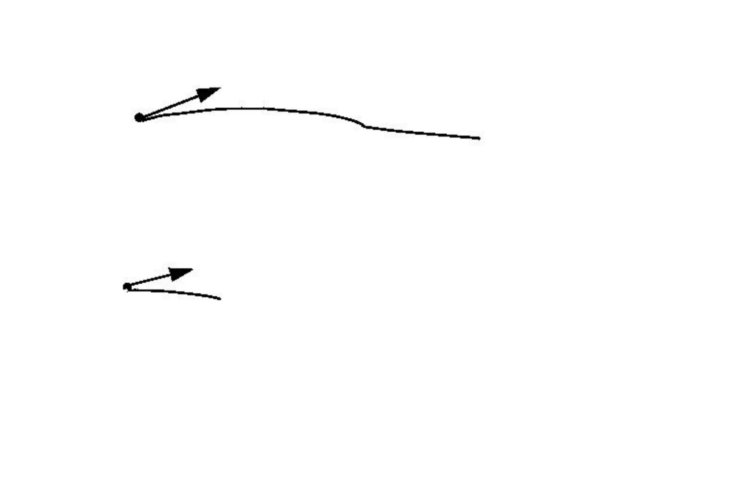

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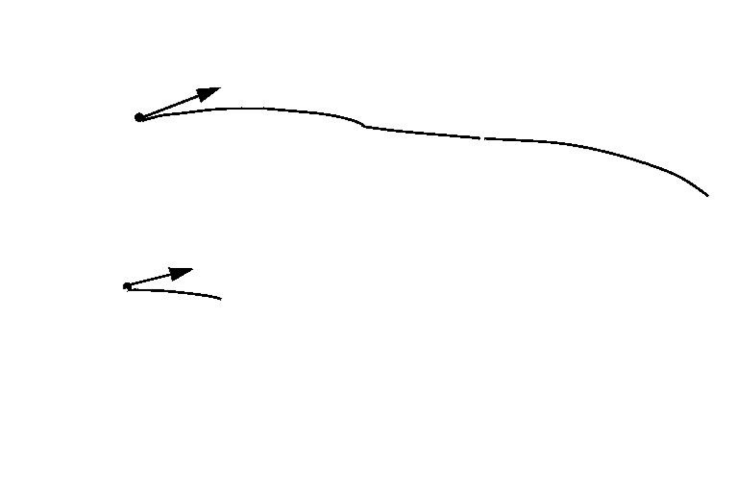

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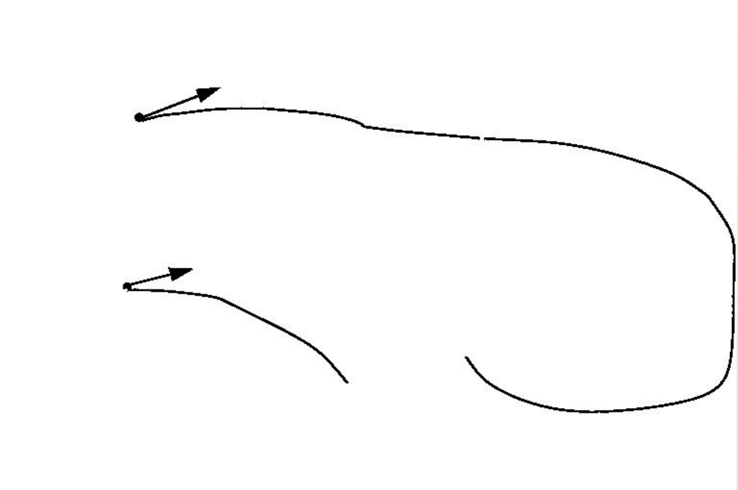

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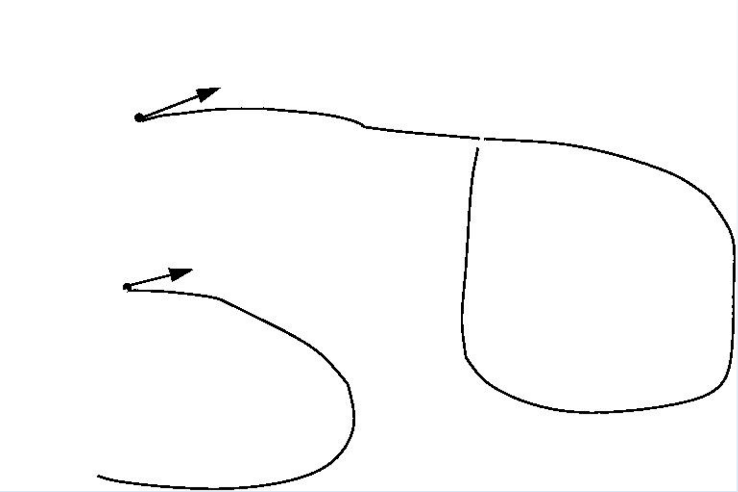

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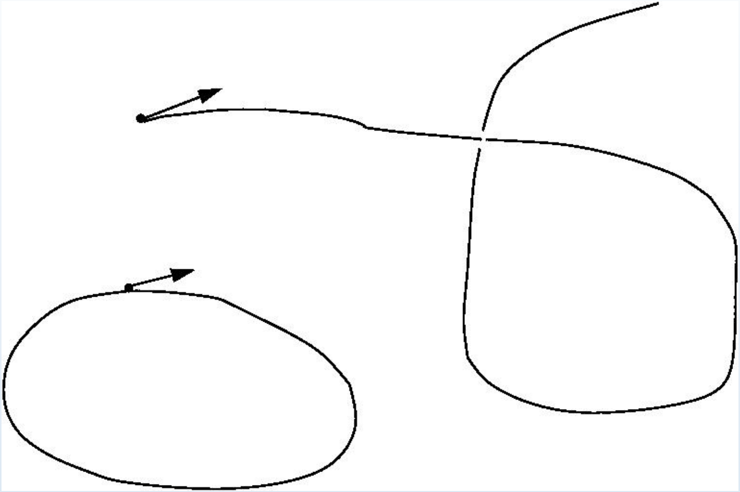

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Can we trust what we get from numerical approximation?

How do we interpret the numerical solutions?

A taxonomy could help

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"A CLASSIFICATION of possible dynamics"
would help to interpret the results

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A "TAXONOMY" of generic well described dynamical behaviors

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Fuzzy information

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Fuzzy information —> better description.

“How to get a *Taxonomy*”

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Looking for the possible *generic* dynamical scenario

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“MODEL OF THE MODELS”

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Looking for the possible *generic* **dynamical scenario**

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“Reality to be understood“

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How to get those model of the models? Example.

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Poincaré and 3 body problem

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Poincaré and 3 body problem

3 bodies interacting under their gravitational attraction.

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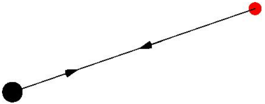

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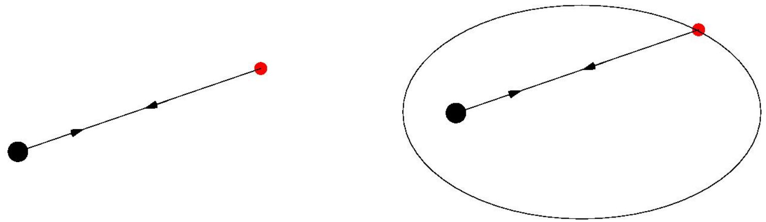

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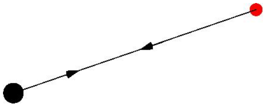

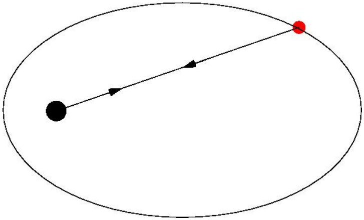

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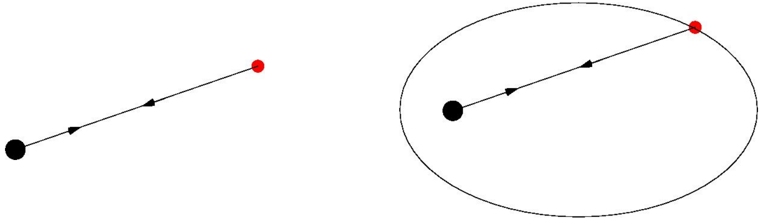

Elliptic motions. Motions are periodic.

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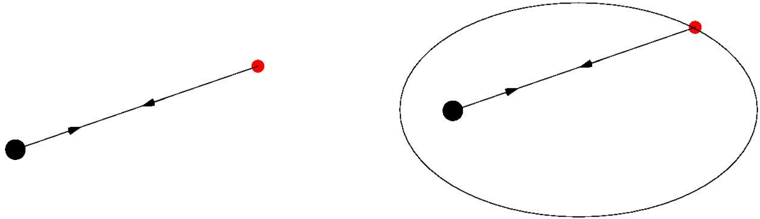

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Poincaré: 3 body problem as a perturbation of the 2 body one.

3 body problem.

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Poincaré observed “complicated trajectories”.

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“...Nothing can give us a better idea of the intricacy of the three-body problem....”

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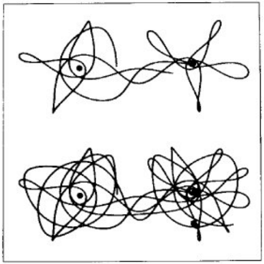

Figure 1. Orbits related to the thee-body problem (Modified from Stewart,

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It can be found in the original problem.

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Homoclinic points: “Points with same past and future"

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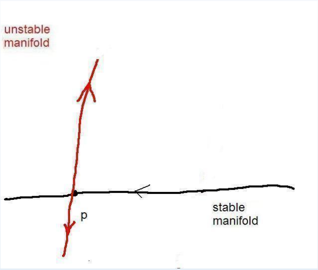

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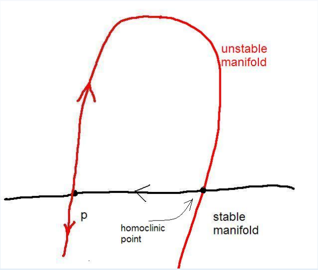

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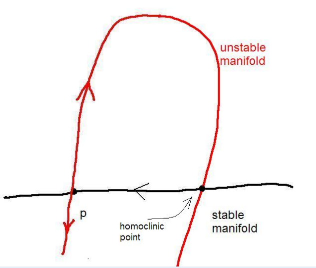

Infinitely many periodic orbits.

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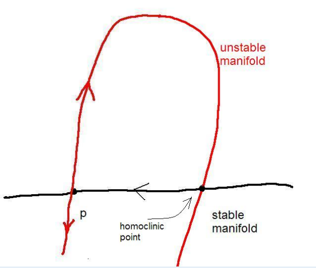

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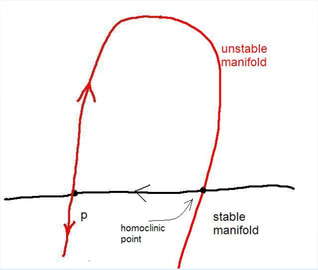

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Chaotic dynamic.

It holds in a robust way.

Homoclic points.

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Appears in the 3 body problem.

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Perturbed pendulum.

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Generic in mechanical problems with at least two degree of freedom

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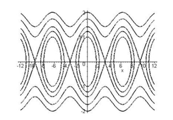

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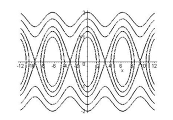

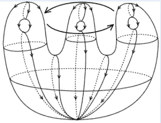

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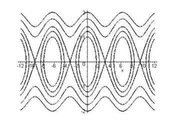

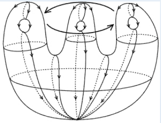

- or there is a transversal homoclinic point.

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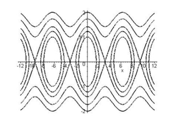
Appears in the 3 body problem.

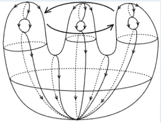
Perturbed pendulum.

Generic in mechanical problems with at least two degree of freedom

Dichotomy

- Either the dynamics is VERY SIMPLE





- or there is a transversal homoclinic point. There are chaotic components.

A Paradigm for generic dynamics

A Paradigm for generic dynamics

A LIST/TAXONOMY OF POSSIBLE DYNAMICAL PHENOMENAS

A Paradigm for generic dynamics

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A LIST OF POSSIBLE MECHANISMS

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BUILD A DICTIONARY/MECHANISMS

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Fuzzy information

A Paradigm for generic dynamics

A LIST/TAXONOMY OF POSSIBLE DYNAMICAL PHENOMENAS

A LIST OF POSSIBLE MECHANISMS

BUILD A DICTIONARY/MECHANISMS

Fuzzy information —> better description.

Coming back to a concrete EAA

Coming back to a concrete EAA

A robot that moves on an Arena (table)

Coming back to a concrete EAA

A robot that moves on an Arena (table)

- starting from a random position, it follows a light turned on in a place also chosen randomly

Coming back to a concrete EAA

A robot that moves on an Arena (table)

- starting from a random position, it follows a light turned on in a place also chosen randomly
- a light is flashed and the robots move into that place where it was flashed.

Dissecting the agent/problem

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- Space/Arena: a two dimensional rectangle where the robot moves;

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- Agent/Robot:
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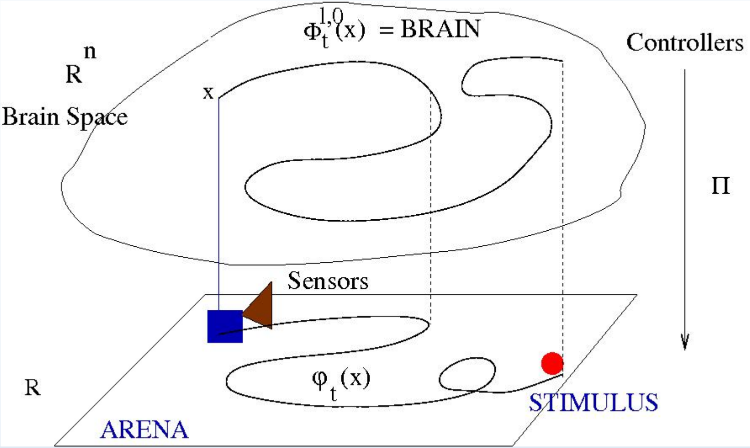
  3. The controllers: A function Π that reads the neural activity and controls the wheels.

Dissecting the agent/problem



More on the “brain”/Neural network

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A parameter family of equations

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$$B = \{X^0, X^l\}_{l \in R}$$

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X^l, equation acting when the light is on at position l (stimulus)

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Trajectories in the brain space

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$\Phi^l : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ solutions of X^l

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Trajectories in the Arena /Projections of brain trajectories

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Trajectories in the Arena /Projections of brain trajectories
$\Psi^l : \Pi(\Phi^l)$ proyections of Φ^l
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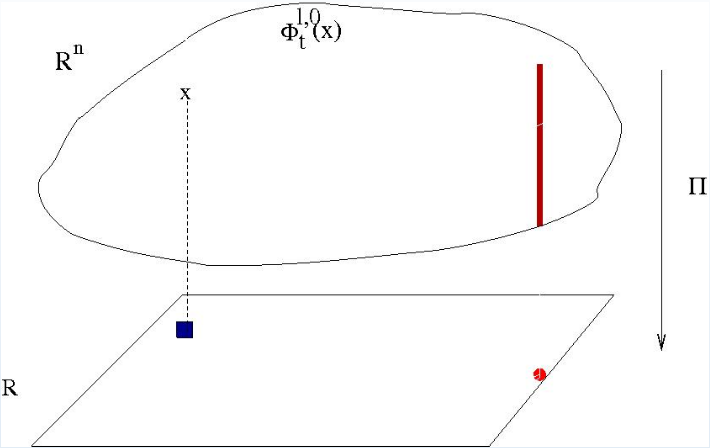
Sketch of trajectories



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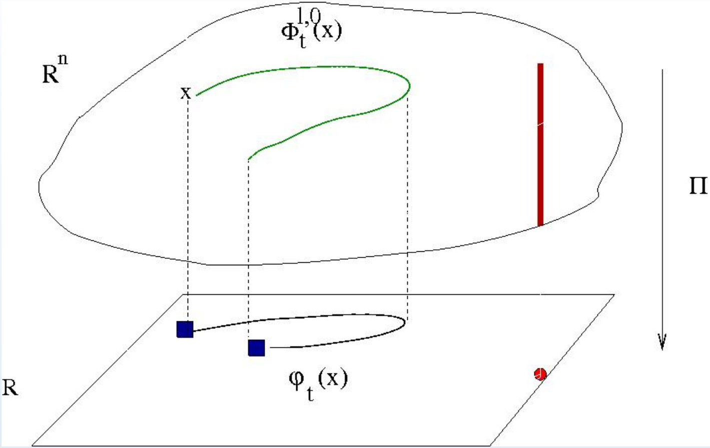

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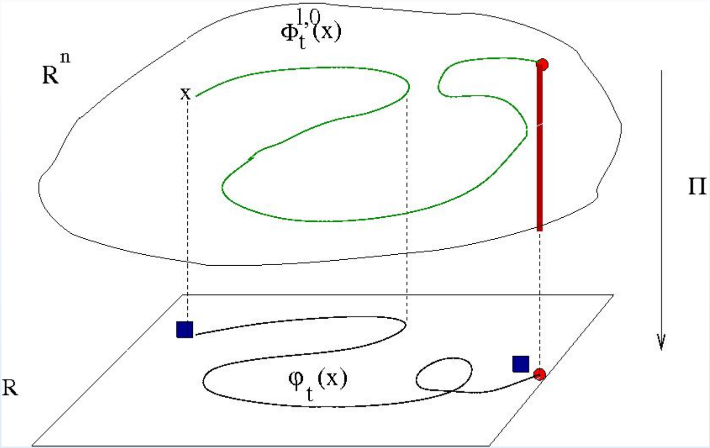

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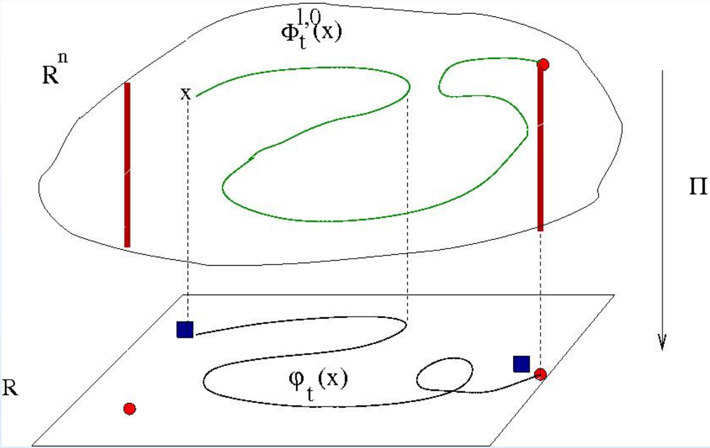

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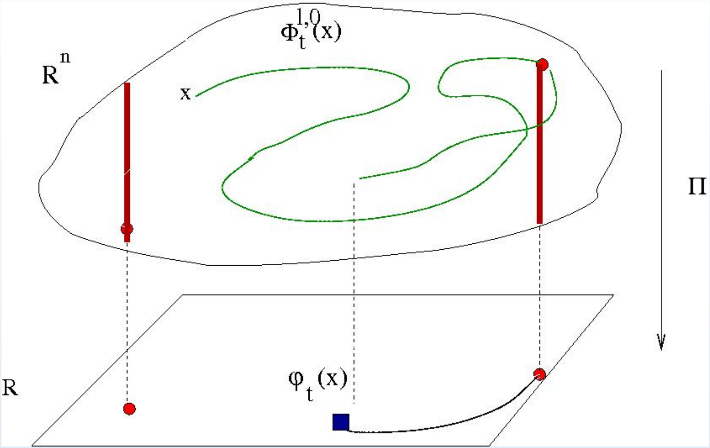

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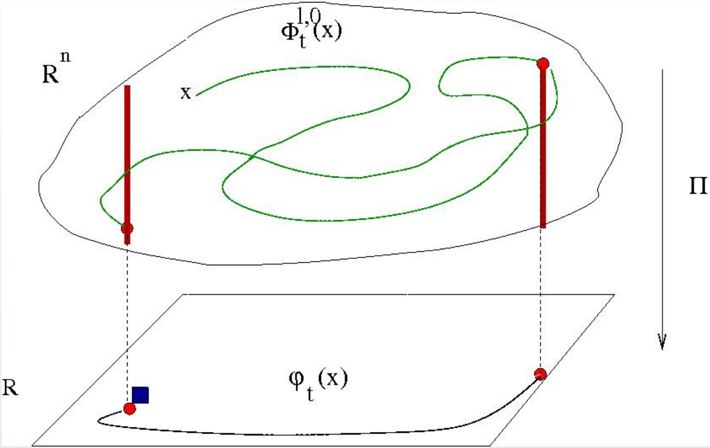
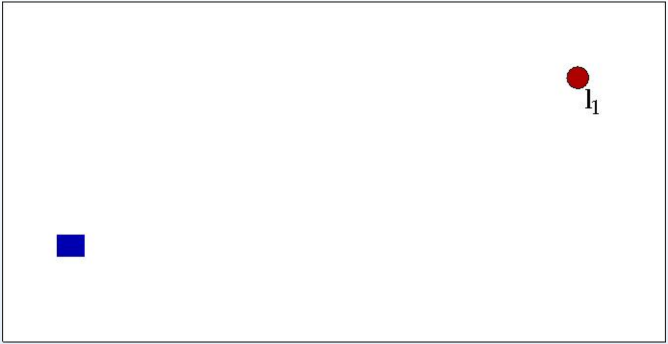


The equation/flow X^l on the Arena

Light is on at position l_1



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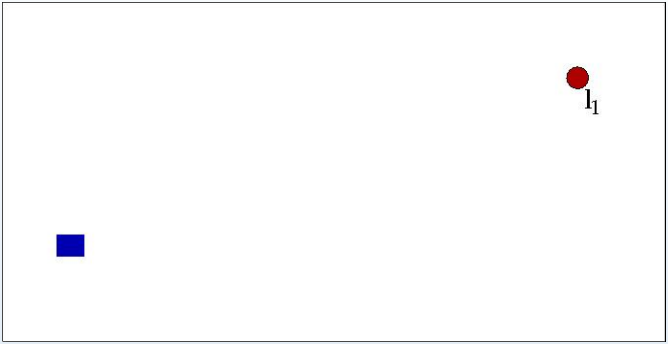

Acts equation/flow $\Psi_t^{l_1}(x) = \Pi(\Phi_t^{l_1}(x))$

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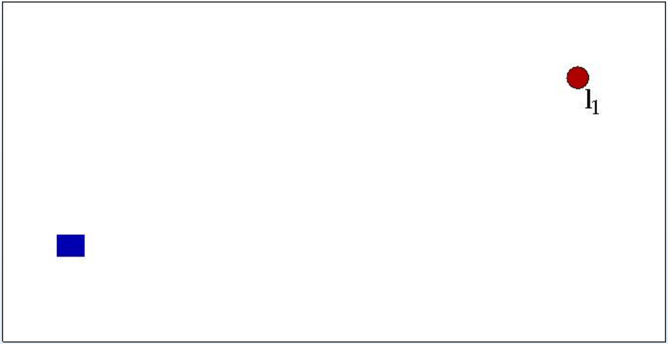

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l_1 is an attractor for Ψ^{l_1}

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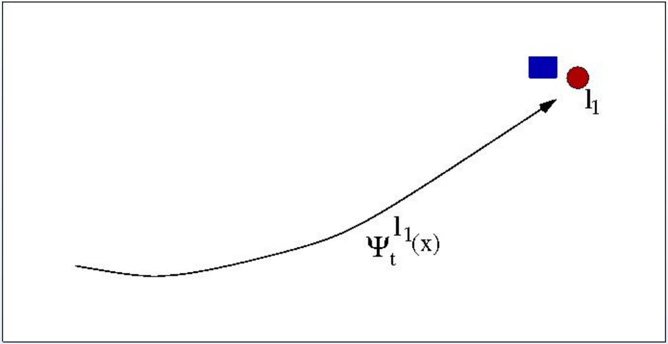

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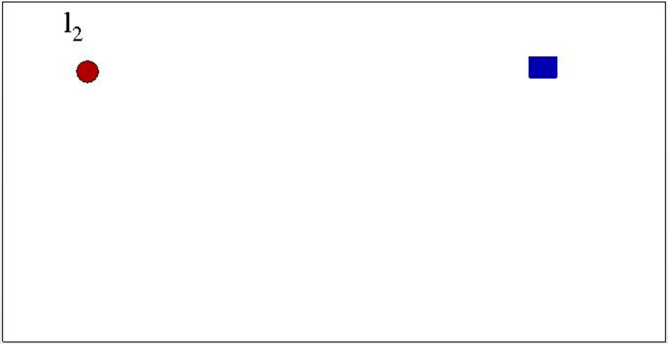
The equation/flow X^l on the Arena

Light is on at position l_2



The equation/flow X^l on the Arena

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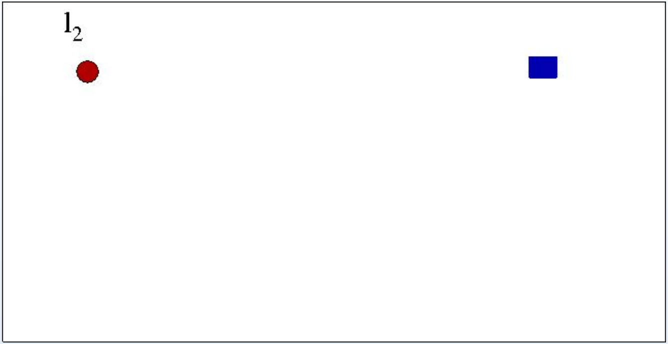

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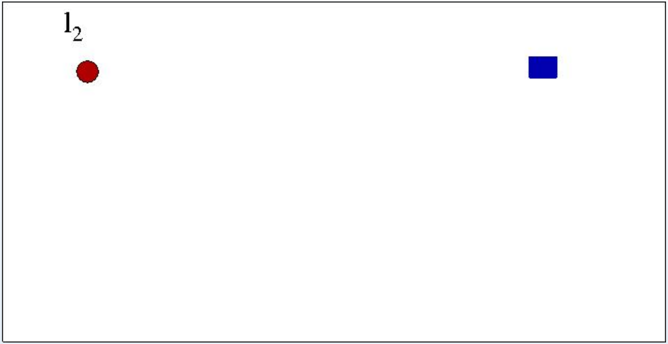

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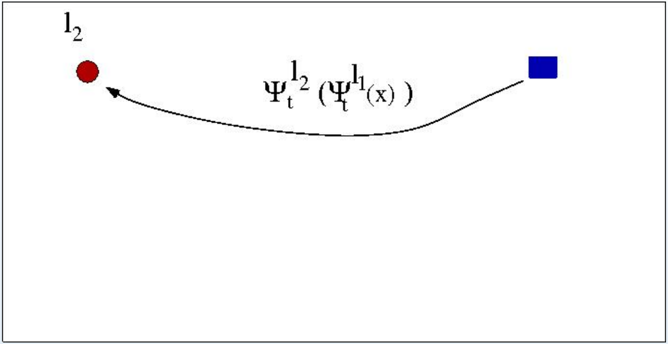

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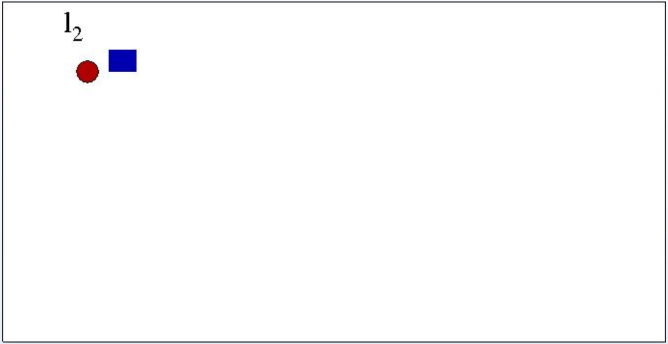

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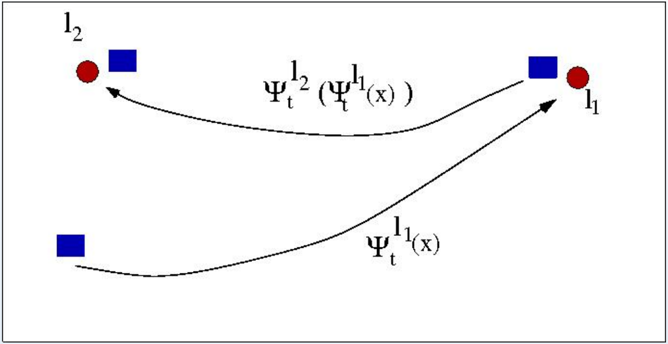

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The equation/flow X^l on the brain space

Light is on at position l_1, l_2

The equation/flow X^l on the brain space

Light is on at position l_1, l_2

$$\Phi_t^{l_i}(x) \to l_i$$

The equation/flow X^l on the brain space

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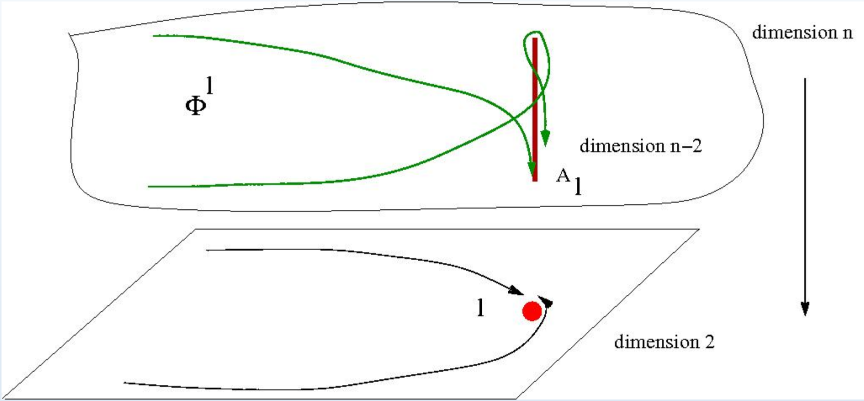

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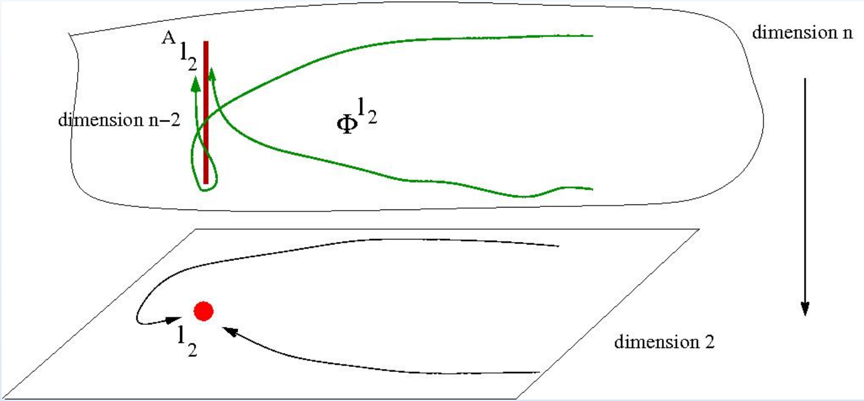

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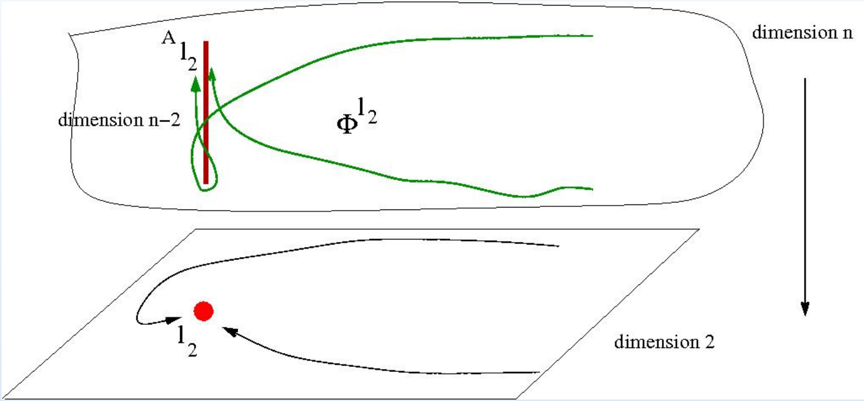


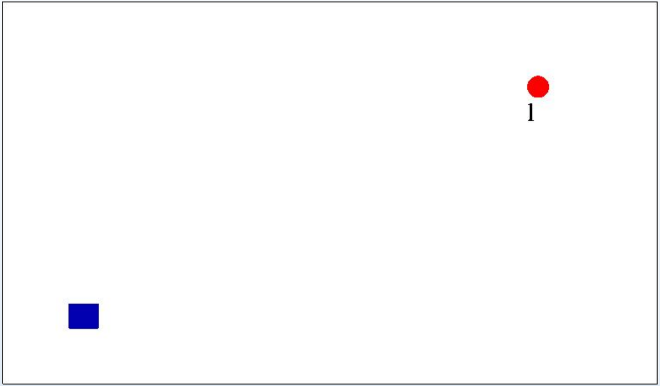
Each Φ^l has a unique attractor A_l that $\Pi(A_l) = l$

The equation/flow X^0 on the Arena

$\Psi^0_t = \Pi(\Phi^0_t)$ has to go to I (any I) in the Arena

The equation/flow X^0 on the Arena

$\Psi^0_t = \Pi(\Phi^0_t)$ has to go to l (any l) in the Arena



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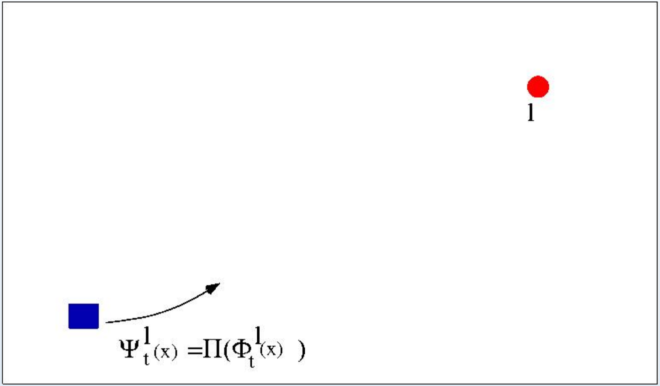

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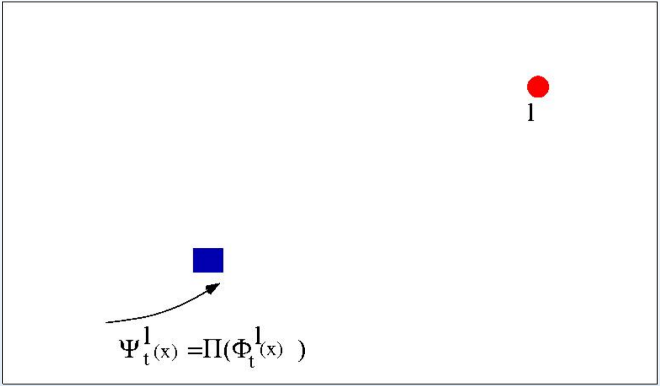

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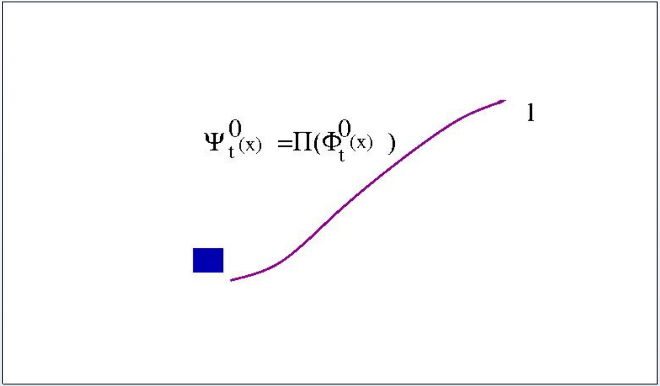

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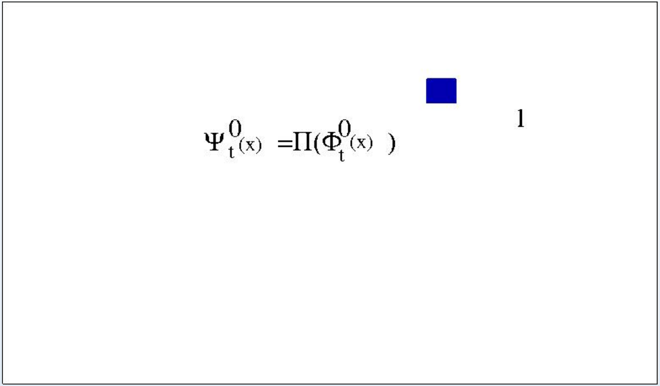

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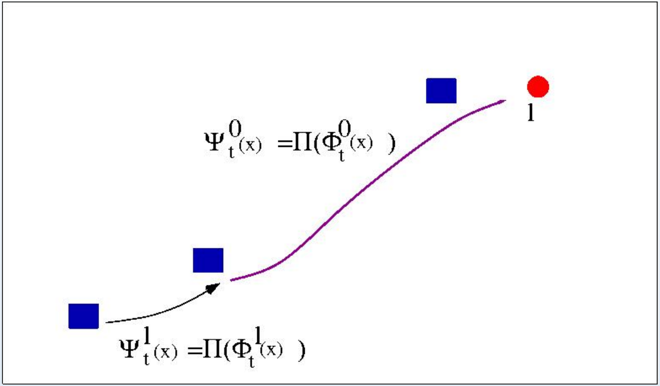

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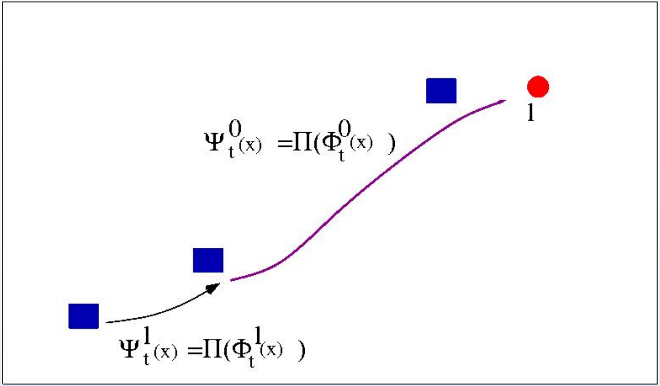


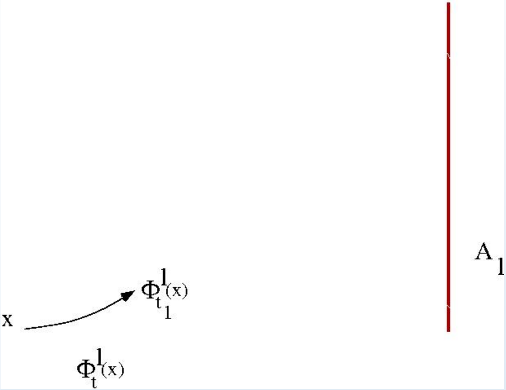
A_l is an attractor for Φ^0 $(\Pi(A_l) = l)$

The equation/flow X^0/Φ^0 on the Brain space

Φ^0_t has to go to A_l (any l) in the Brain space

The equation/flow X^0/Φ^0 on the Brain space

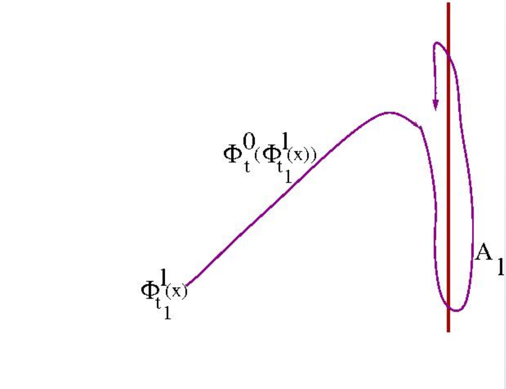
Φ^0_t has to go to A_l (any l) in the Brain space



First acts Φ^l_t

The equation/flow X^0/Φ^0 on the Brain space

Φ_t^0 has to go to A_l (any l) in the Brain space



Later acts Φ_t^0

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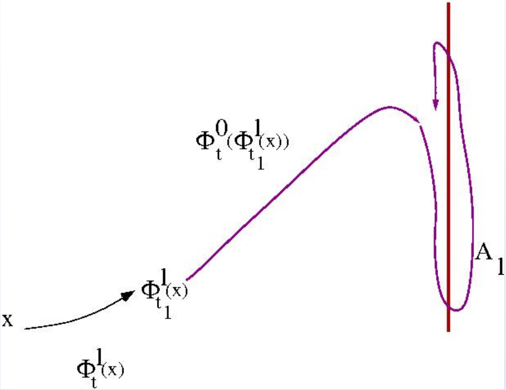
Φ_t^0 has to go to A_l (any l) in the Brain space



Φ^0 has an attractor at A_l that $\Pi(A_l) = l$

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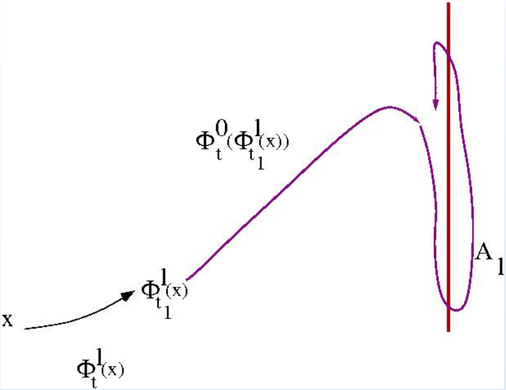
Φ^0_t has to go to A_l (any l) in the Brain space



Φ^0 has an attractor at A_l that $\Pi(A_l) = l$

So, Φ^0 has "many" attractors

Infinitely many attractors for X^0/Φ^0 on the Brain space

Infinitely many attractors for X^0/Φ^0 on the Brain space

So, Φ^0 has “many“ attractors

Infinitely many attractors for X^0/Φ^0 on the Brain space

So, Φ^0 has “many“ attractors

As many as places where the robot has to go

Infinitely many attractors for X^0/Φ^0 on the Brain space

So, Φ^0 has “many“ attractors

As many as places where the robot has to go

But if you are in an attractor, how do you move to others?

Infinitely many attractors for X^0/Φ^0 on the Brain space

So, Φ^0 has “many“ attractors

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How to jump to the next attractor

Infinitely many attractors for X^0/Φ^0 on the Brain space

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How to jump to the next attractor

A recalling from dynamics: Basin of attraction

Infinitely many attractors for X^0/Φ^0 on the Brain space

So, Φ^0 has “many“ attractors

As many as places where the robot has to go

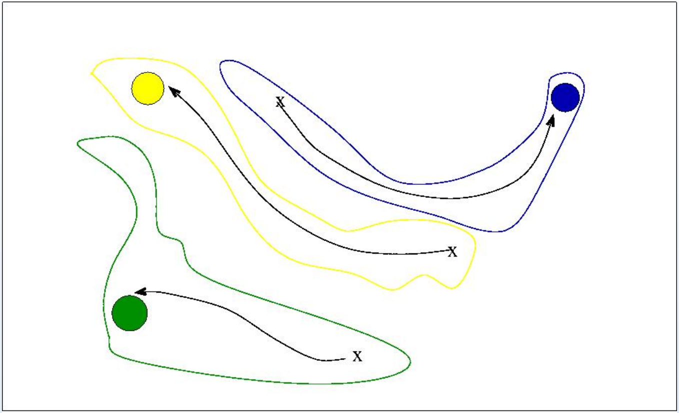
But if you are in an attractor, how do you move to others?

How to jump to the next attractor

A recalling from dynamics: Basin of attraction

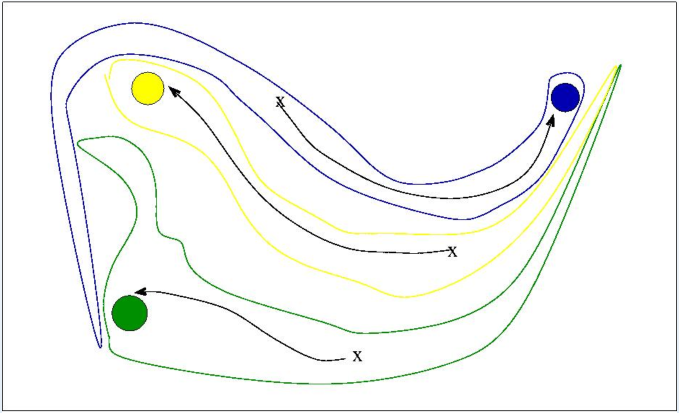
X^I, Φ^I **is kicking out with a small STIMULUS**

Infinitely many attractors for X^0/Φ^0 on the Brain space



Attractors and its basin of attractions

Infinitely many attractors for X^0/Φ^0 on the Brain space



Attractors and its basin of attractions

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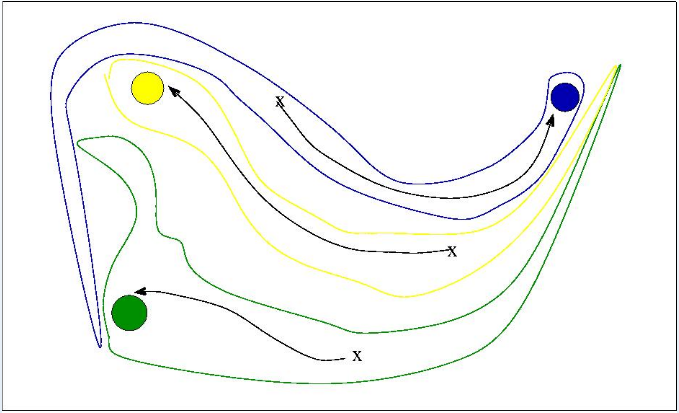
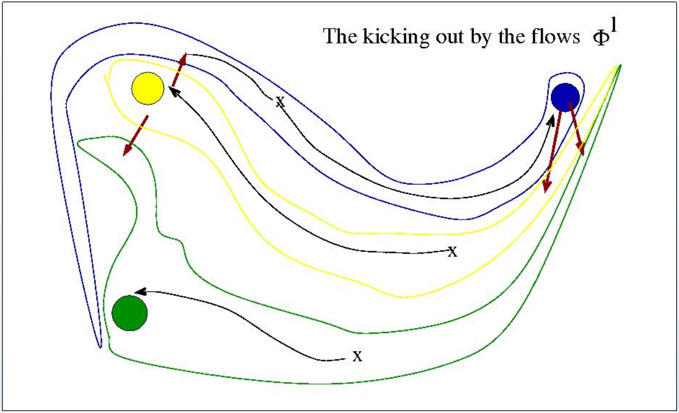


Attractors and its basin of attractions

Intertwining basin of attraction

Infinitely many attractors for X^0/Φ^0 on the Brain space



Attractors and its basin of attractions

Intertwining basin of attraction

Flows Φ^l moves between basins

Is it possible to have such phenomena?

Yes. WILD DYNAMICS.

Is it possible to have such phenomena?

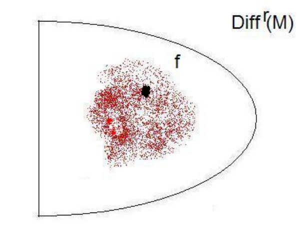
Yes. WILD DYNAMICS.

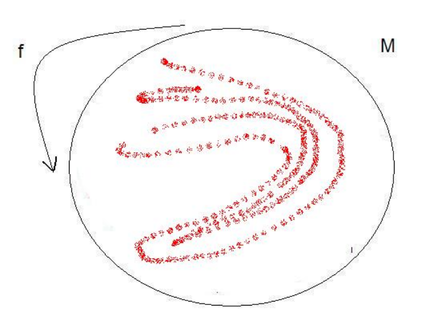
Infinitely many attractors appearing and dissapearing

Is it possible to have such phenomena?

Yes. WILD DYNAMICS.

Infinitely many attractors appearing and dissapearing





Is it possible to have such phenomena?

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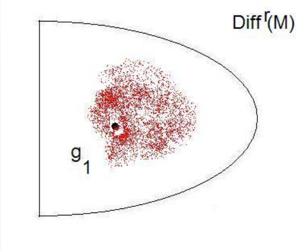

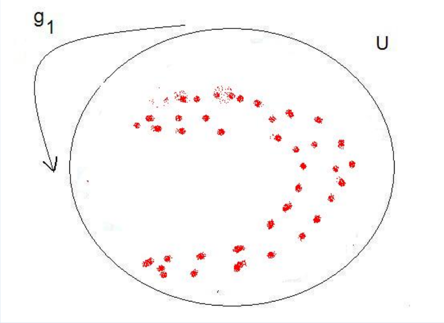

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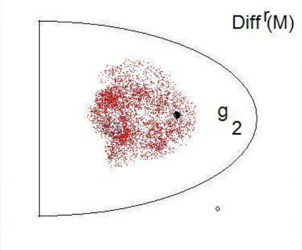

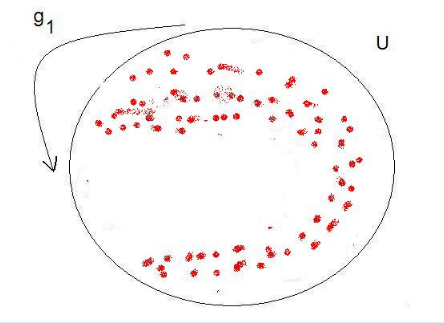

Mechanisms for wild dynamics?

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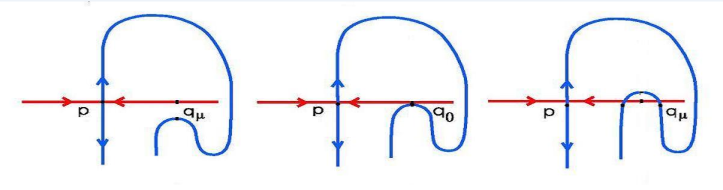

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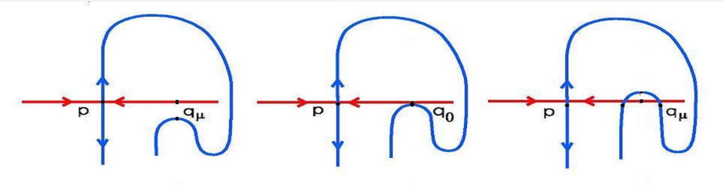

Residual sets of systems having infinitely many attractors.

Properties of Wild Dynamics

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power law properties

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Many of the features that appears in the edge of chaos

.

And the Smart Data?

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Is this approach relevant for understanding EAA?

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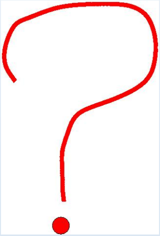

So?

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We need a model to start

So?

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At least a TOY

So?

We need a model to start

At least a TOY and start to play

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being pragmatic